Area Under a Stream Hydrograph

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Time:
(1-2) 45 minutes classroom sessions, depending on how in-depth you go with the subject

Level:
AP Calculus

Goals:
This lesson plan will give students an opportunity to learn how to use hydrographs to approximate the amount of discharge from a river.

Objectives:
Students will be able to:

1. Approximate the area under a curve using left and right hand Riemann sums.

2. Approximate the area under a curve using trapezoidal approximations.

Materials listed with each individual activity.

This lesson plan describes methods for calculating total stream flow, or discharge, by calculating the area under a curve.

Correlations to Core Curriculum:

AP Calculus

- Numerical approximations to definite integrals:
    - Use of Riemann sums (using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.

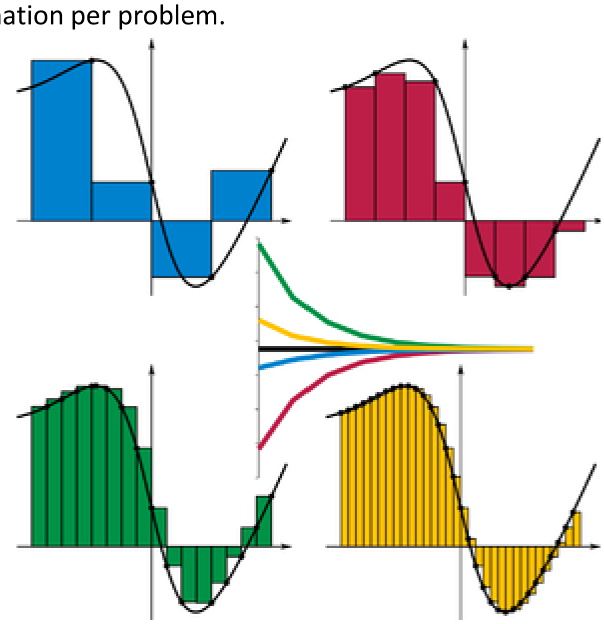
Background Information:

<u>Hydrographs</u>

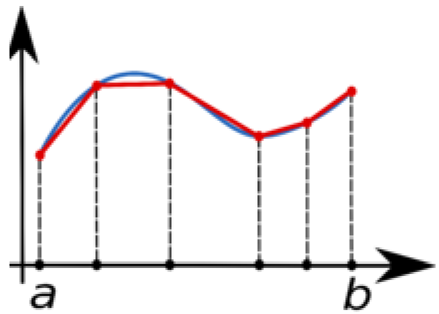
A hydrograph shows the changes in a river's discharge over a period of time. There are several factors that can affect the shape of a hydrograph such as: topography, land-use, duration of rainfall, seasons, vegetation type, river conditions, soil permeability, etc.

<u>Riemann sums</u>

Below in blue is an example of right rectangles and in yellow are left rectangles. In red and green are a mixture or left and right rectangles, but in this class, we will stick with only doing one type of approximation per problem.



Trapezoidal Approximations Example



Factors Affecting River Discharge

Rock and soil type

- Permeable rocks and soils (such as sandy soils) absorb water easily, so surface run-off is less common.
- Impermeable rock and soils (such as clay soils) are more closely packed. Rainwater can't infiltrate as quickly, resulting in runoff and water reaching the river more quickly.
- Pervious rocks (like sandstone and limestone) allow water to pass through spaces between the rock particles.

Land use

- In urban areas, surfaces like roads are impermeable – water can't soak into the ground. Instead, it runs into drains, gathers speed and joins rainwater from other drains – eventually spilling into the river.
- In rural areas, plowing up and down (instead of across) hillsides creates channels which allow rainwater to reach rivers faster increasing discharge.
- Deforestation means less interception, so rain reaches the ground faster. The ground is likely to become saturated and surface runoff and erosion will increase.

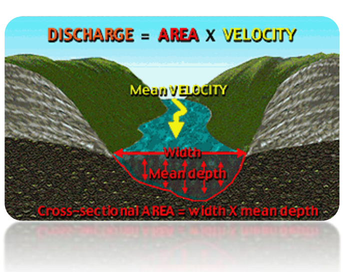
Rainfall

- The amount and type of rainfall will affect a river's discharge.
- Antecedent rainfall is rain that has already happened. It can mean that the ground has become saturated. Further rain will then flow as surface run-off towards the river.
- Heavy continual rain, or melting snow, means more water flowing into the river.

Did you know?

Discharge is the volume of water moving down a stream or river per unit of time, commonly expressed in cubic feet per second or gallons per day.

http://water.usgs.gov/edu/streamflow2.html



Topography

- Steep slopes mean that rainwater is likely to run straight over the surface before it can infiltrate. On more gentle slopes infiltration is more likely.

Weather conditions

- Hot dry weather can create a crust on top of soil, so that when it rains the water will run off of the surface, straight into the river.
- High temperatures increase evaporation rates from water surfaces, and transpiration from plants – reducing discharge.
- Long periods of extreme cold weather can lead to frozen ground, preventing water from infiltrating.

http://www.acegeography.com/factors-affecting-river-discharge.html

Materials:

Supplies –

- Area Approximation worksheet (At the end of the lesson plan, print 1 per student)
- Hydrograph Assessment (At the end of the lesson plan, print 1 per students)
- Pencils

Equipment--

- Whiteboard, Dry Erase Markers
- Document Camera (optional)

Activities:

Day 1 --

Engage (5 minutes) – Discuss the definition of river discharge, and the many different factors that affect river discharge. Among other things, you may want to discuss the effects of rock and soil type, land use, rainfall, weather conditions, and topography. (More information can be found in the 'background information' section of this lesson plan.)

Explain that a special type of graph called a hydrograph is used to show river discharge over periods of time. Show a hydrograph to students, explaining a basic description of how to read the hydrograph. Tell students that their challenge will be to approximate the area under the curves on the hydrograph. They should try to get their estimations to be as accurate as possible.

Tell the class that they will be working with partners to determine an assigned area under a curve. Either assign or let the class choose partners. Assign half of the class partnerships to work on the 'red curve,' and the other half of the partnerships to work on the 'blue curve.'

Explore (30 minutes) – Pass out the 'Hydrograph Discharge' worksheet (attached at the bottom of this lesson plan). Working in pairs, have the students discuss how to approximate the amount of discharge (area under the curve). They will then use whatever method they determine to approximate the amount of discharge.

The red and blue students will record their respective approximations on the whiteboard.

Call on several groups to explain how they approximated the amount of discharge. Discuss the differences among the red approximations and then the differences among the blue approximations. Did one group have a better approximating method than another group? Which approximation is more accurate?

Did you know?

You can visit USGS Water Watch to see current streamflow in your area! You can see if your water level is average, below average, or even above average.

Teach the students an approximation method called Riemann sums. Teach that it is possible to approximate the area under curves by making rectangles where either the right or left side of the rectangle lands on the curve. The more rectangles that you make, the more accurate your approximation will be. Once you construct the rectangles, you find the area of each rectangle by multiplying the length and width ($A=lw$) and then add all the areas together. The approximation may be an over approximation or an under approximation depending upon the way the rectangles are drawn.

Have the students work on either their red or blue graph doing the Riemann approximations. Half of the reds and half of the blues will do right approximations and the other half of each respective group will do left approximations. Assign each pair a different number of rectangles to create such as: 2, 4, 6, or 8. Have the students then record the results on the whiteboard.

If needed, stop here and continue with the lesson during the next class period

Teach the students a third method of approximating the area under a curve using trapezoidal approximations. Instead of making rectangles, you make trapezoids with two points of the trapezoid landing on the curve. You then calculate the area of the individual trapezoids -- ($A=h/2\ (b_1 + b_2)$) and add them together. This approximation is even better than that of the left and right hand Riemann sums. (You could teach midpoint Riemann approximations before the trapezoidal approximation, do the trapezoidal approximation first because you use the same endpoints that were determined with the left and right hand Riemann approximations.)

Have the students then calculate their area under their red or blue graph. Have each pair do the same number of trapezoids as they did rectangles: 2, 4, 6, or 8. Have the students then record the results on the whiteboard. What do they notice about the results for the red and blue graphs? (The approximations for the two graphs should be close because they have the same area under the curves even though they have different shapes.)

Explain (10 minutes) – Discuss why hydrographs (and a knowledge of how they work) are important in taking care of our watersheds. Discuss with the students stewardship of the land. Who is/should be responsible for taking care of public lands? How can litter affect the discharge of a river? When you go camping or hiking, how should you leave the environment? What would happen if everyone ignored the litter and no one was willing to pick up junk someone else left behind? What can you do to be a good steward?

Did you know?
A number of factors (known as drainage basin controls) influence the way in which a river responds to precipitation and have an effect on the shape of the hydrograph.

http://www.bbc.co.uk/scotland/education/int/geog/rivers/hydrographs/

Day 2 –

If needed, finish the lesson on trapezoid estimation. Allow the students to discuss their favorite and most accurate way of finding the area. If extra time is available, have students do some practice problems to become more comfortable with the new techniques of finding area under curves that they have learned. You may also choose to give the assessment today if it is the end of your unit. The assessment for this activity is attached at the bottom of the document.

Assessment:
An assessment has been created and attached to the bottom of this lesson plan.

Extensions:
- Have a career specialist in your area give a presentation on the importance of using hydrographs accurately in the real world.
- Have students create their own 'imaginary river' situation. Determine velocity and area for 15 years (one measurement per year). Students can figure out the discharge each year, and create their own hydrographs.
- Use Riemann sums to calculate the cross sectional area of a stream, and multiply that by the stream velocity to measure stream flow.

Resources:

Books
- *River Discharge Prepared for the Use of Engineers and Students* by John Clayton Hoyt

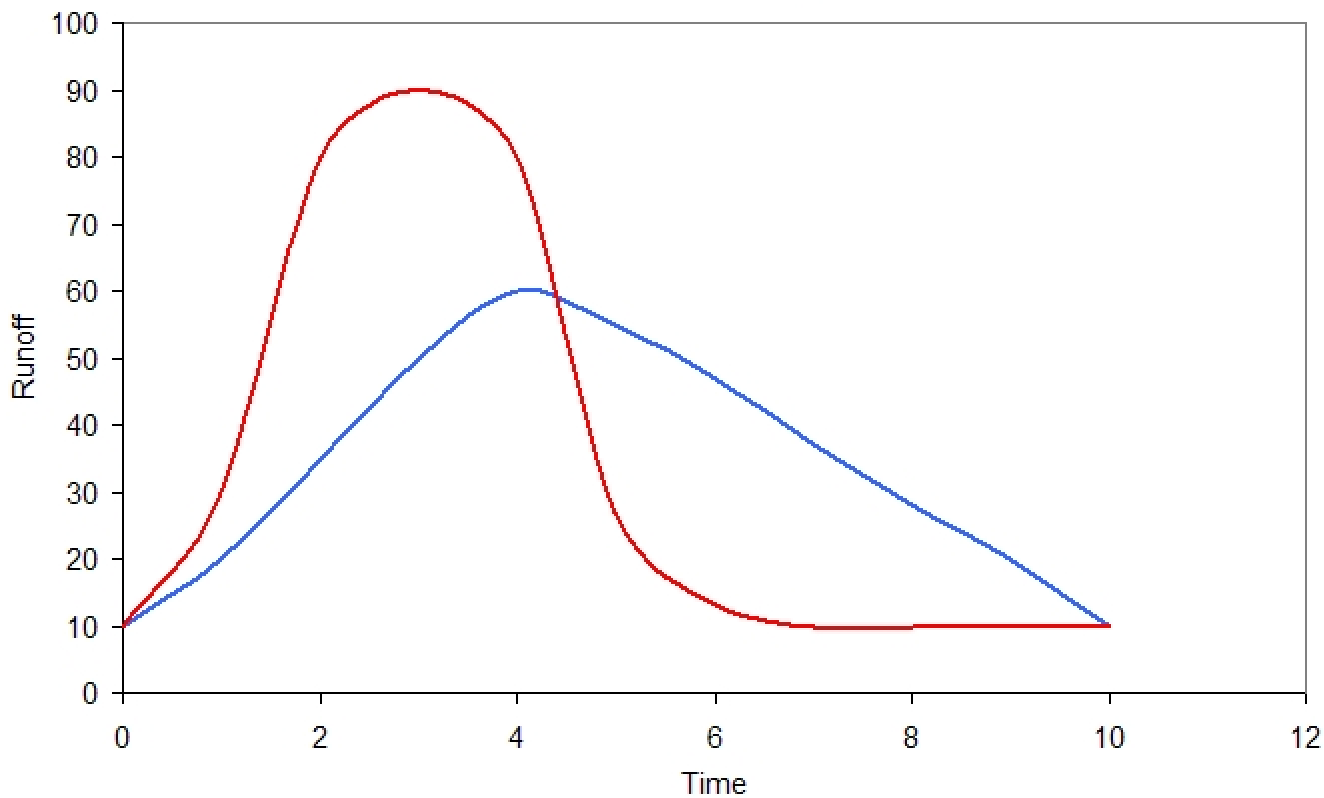
- *Utah Master Naturalist Watersheds Textbook* *http://extension.usu.edu/utahmasternaturalist/files/uploads/UMNP_Watersheds_Text.pdf*

Websites

- Factors affecting river discharge -- http://www.acegeography.com/factors-affecting-river-discharge.html
- How discharge is measured -- http://water.usgs.gov/edu/streamflow2.html
- Current streamflow in your area -- http://waterwatch.usgs.gov/index.php?id=ww

Name: ______________________________________ Area Approximation

Circle the color you are assigned to work with: RED or BLUE. Only do calculations based on what you were assigned.

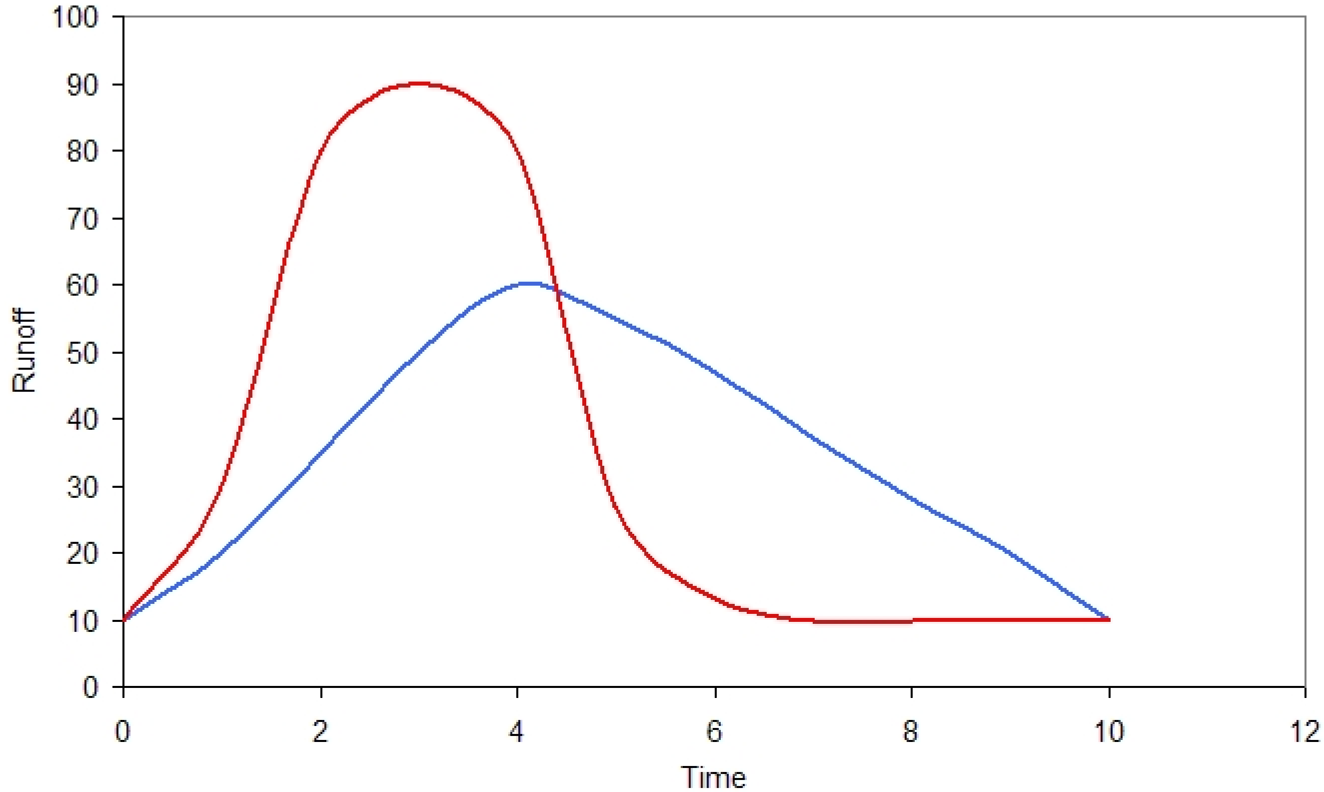


1. How could you figure out the amount of runoff (discharge) for your assigned graph?

2. Use your method to approximate the total runoff.

3. Is your answer an over approximation or an under approximation?

Now use Riemann sums to approximate the area under your assigned curve. Circle the number of rectangles you were assigned to use: 2, 4, 6, or 8.



4. Approximation using left Riemann sums:

5. Approximation using right Riemann sums:

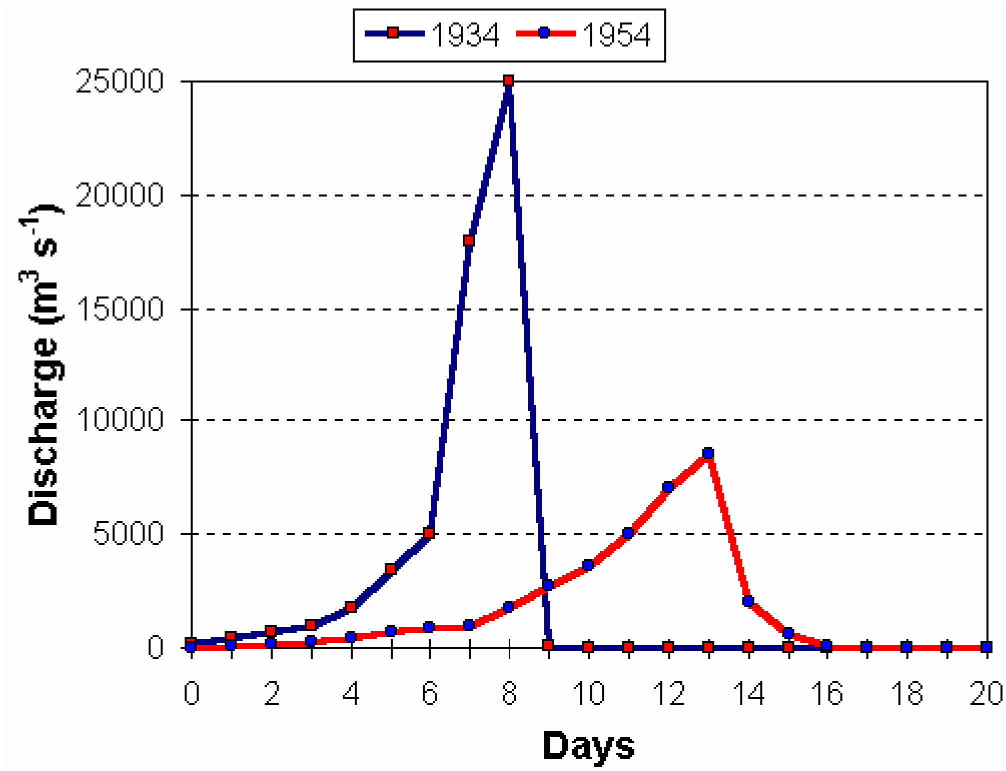
6. Now use trapezoids to approximate the area under your assigned curve. Make the same number of trapezoids as you did rectangles.

7. Comparing the class results for the total runoff for the red and blue graphs, what do you notice?

8. What are your responsibilities for stewardship of the land? How will you take care of the land?

Hydrograph Assessment

Name: ____ Date: ________________



Use the graph above to approximate the discharge in 1934 using left and right Riemann sums with 4 rectangles, and a trapezoidal approximation using 4 trapezoids. Show your work.

1. Left Riemann sum:

2. Right Riemann sum:

3. Trapezoidal approximation:

4. Which approximation do you think is more accurate and why?